

Casimir effect of scalar and Yang-Mills theories in the infrared limit

Marco Frasca*

via Erasmo Gattamelata, 3, 00176 Roma, Italy[†]

(Dated: September 1, 2011)

Abstract

We study both massless scalar and Yang-Mills field theories in the deep infrared in presence of a simple boundary. We can show, with the help of the recent scenario emerging from studies on the propagators, that the presence of a mass gap makes the Casimir contribution exponentially small as should be expected. The existence of a trivial infrared fixed point shared by both theories makes the computation as simple as the free particle case.

*Electronic address: marcofrasca@mlink.it

[†]Homepage: <http://marcofrasca.wordpress.com>

I. INTRODUCTION

Vacuum in quantum field theory is known to not be inert. The most striking evidence of this was put forward in 1945 by Casimir [1] providing an example of a macroscopic effect produced by quantum fluctuations. Due to the smallness of the involved elements, an experimental proof of the Casimir effect has been obtained quite recently (see [2] for a review).

Casimir effect is an example of a non-trivial behavior of a quantum field theory due to the presence of boundary conditions that somehow modify the behavior of the free space case. This kind of question is interesting *per se*. But while in the ultraviolet regime we have techniques able to cope with this kind of problems, in the opposite limit, the infrared regime, such techniques were generally lacking. It is important to note that the behavior at different energy regimes of a quantum field theory could result quite different depending on the structure of vacuum. So, we know that at high energies, QCD behaves as an almost free theory and asymptotic states of the Yang-Mills field are massless gluons. On the other side, at lower energies, a Yang-Mills theory displays a mass gap yielding a very different behavior in this case. E.g. this could be inherent to a non-trivial non-perturbative vacuum seen as an instanton liquid [3].

Studies of quantum field theories in the infrared limit have had an important rebirth in this last two decades by the use of Dyson-Schwinger equations [4, 5] and by the improvement in computing resources that made possible to analyze Yang-Mills theory on larger lattices. The theoretical proposal to solve the infinite hierarchy of Dyson-Schwinger equations was a truncation scheme that, for the Landau gauge, produced a gluon propagator going to zero and a ghost propagator running to infinity faster than the free case with momenta going to zero. Such a scenario was previously devised by Daniel Zwanziger [6] providing also a criterion for color confinement then dubbed Gribov-Zwanziger criterion. Initial lattice computations seemed to support these conclusions even if no bending toward zero of the gluon propagator was ever observed at lower momenta. People generally thought that was just a matter of volumes and, increasing computational resources, things should have changed.

The breakthrough came out in 2007. At the Lattice 2007 Conference in Regensburg, three groups presented their results with huge volumes arriving to such a significant value as $(27fm)^4$ [7–9]. The shocking result was that the scenario devised since then, generally

accepted as correct, was not describing the situation seen on the lattice: The gluon propagator was reaching a plateau at lower momenta with a finite non-zero value in zero and the ghost propagator was behaving as that of a free massless particle. All in all, the running coupling was seen to bend clearly toward zero without evidence of a non-trivial fixed point as was expected instead.

In the eighties, a classical paper by Cornwall [10] showed that indeed the gluon propagator showed a dynamical mass, dependent on momenta, that in the zero limit reaches a finite constant value. With the emerging of techniques to solve Dyson-Schwinger equations in the nineties, it was straightforward to try to solve them numerically. This numerical approaches showed that Cornwall view is indeed correct [11] but this paper displayed also all the scenario seen since 2007 on lattice computations. So, a research line developed producing an in-depth theoretical and numerical analysis of Dyson-Schwinger equations supporting the current view [12–21].

The idea in this paper is to start from this situation, also supported by theoretical arguments, to analyze the behavior of the vacuum in the infrared limit. The best way to see a non-trivial behavior is through the analysis of the deep infrared limit with simple boundary conditions. As we will see, this can be accomplished yielding the result, somewhat expected with a mass gap, that the Casimir contribution is exponentially damped both for scalar and Yang-Mills theories.

The paper is so structured. In sec.II we discuss the theoretical approach to treat quantum field theories in the infrared limit. In sec.III we compare our results with lattice computations and numerical Dyson-Schwinger equations showing how these strongly support our conclusions and providing a theoretical framework to perform our computations. In sec.IV we present our main results on the Casimir contribution. Finally, in sec.V we give our conclusions.

II. INFRARED LIMIT

Infrared physics can be studied through perturbation techniques as already shown in [22–24] much in the same way as it is seen in a weak coupling case. The results appear to be quite enlightening producing explicit analytic solutions both for the quantum and the classical cases in the limit of a bare coupling taken to go formally to infinity. This

approach can be seen as a substantially improved version of the approach devised at the end of seventies by Carl Bender and others [25, 26]. The relevant result useful for our aims is that both the scalar and Yang-Mills theories display a trivial infrared fixed point making really easy to determine the effective potential in this limit.

A. Scalar field theory

For our aims it is enough to consider the following generating functional of a scalar field

$$Z[j] = N \int [d\phi] \exp \left\{ i \int d^4x \left[\frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4 + j\phi \right] \right\}. \quad (1)$$

One can formally rescale space-time coordinates as $x \rightarrow \sqrt{\lambda}x$ and the functional can be rewritten as

$$Z[j] = N \int [d\phi] \exp \left\{ i \int d^4x \left[\frac{1}{2\lambda}(\partial\phi)^2 - \frac{1}{4}\phi^4 + \frac{1}{\lambda}j\phi \right] \right\}. \quad (2)$$

We note here that j is an arbitrary function and can be rescaled as $j/\lambda \rightarrow j$. So, at this stage, we can perform an expansion

$$\phi = \sum_{n=0}^{\infty} \lambda^{-n} \phi_n \quad (3)$$

producing the following terms into the action

$$S_0 = \int d^4x \left[\frac{1}{2}(\partial\phi_0)^2 - \frac{1}{4}\phi_0^4 + j\phi_0 \right] \quad (4)$$

$$S_1 = \int d^4x \left[\partial\phi_0\partial\phi_1 - \phi_0^3\phi_1 + j\phi_1 \right] \quad (5)$$

$$S_2 = \int d^4x \left[\frac{1}{2}(\partial\phi_1)^2 - \frac{3}{2}\phi_0^2\phi_1^2 + \partial\phi_0\partial\phi_2 - \phi_0^3\phi_2 + j\phi_2 \right]. \quad (6)$$

From these, we are a step away from a proof of triviality of this theory in the infrared limit. Indeed, we just note that this functional demands, undoing the rescaling, to solve the equation

$$\partial^2\phi_0 + \lambda\phi_0^3 = j \quad (7)$$

in the limit $\lambda \rightarrow \infty$. Using this equation, we get a functional

$$Z[j] \approx e^{i \int d^4x \left[\frac{1}{2}(\partial\phi_0)^2 - \frac{\lambda}{4}\phi_0^4 + j\phi_0 \right]} \int [d\phi_1] e^{i \frac{1}{\lambda} \int d^4x \left[\frac{1}{2}(\partial\phi_1)^2 - \frac{3}{2}\lambda\phi_0^2\phi_1^2 \right]} \quad (8)$$

where we note that the relation $\phi_0 = \phi_0[j]$ is hidden inside the functional integral. To work out this relation, we note that we can solve eq.(7) by taking the following approximation

[27, 28]

$$\phi_0 = \mu \int d^4x' \Delta(x - x') j(x') + \delta\phi \quad (9)$$

being

$$\partial^2 \Delta(x - x') + \lambda [\Delta(x - x')]^3 = \frac{1}{\mu} \delta^4(x - x'). \quad (10)$$

with μ an arbitrary constant with the dimension of energy. This constant will define the mass spectrum of the theory as we will see below. One gets

$$\begin{aligned} \delta\phi = & \mu\lambda \int d^4x' \Delta(x - x') \left\{ \mu \int d^4x'' [\Delta(x' - x'')]^3 j(x'') \right. \\ & \left. - \mu^3 \left[\int d^4x'' \Delta(x' - x'') j(x'') \right]^3 \right\} + \dots \end{aligned} \quad (11)$$

and higher order corrections can be obtained by iteration. This appears a functional expansion in the current j that is consistent with the large coupling approximation. The propagator can be computed exactly [22, 29] and can be expressed in the following form

$$\Delta(p) = \sum_{n=0}^{\infty} \frac{B_n}{p^2 - m_n^2 + i\epsilon} \quad (12)$$

being

$$B_n = (2n + 1) \frac{\pi^2}{K^2(i)} \frac{(-1)^n e^{-(n+\frac{1}{2})\pi}}{1 + e^{-(2n+1)\pi}}, \quad (13)$$

with $K(i) = \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1+\sin^2\theta}} \approx 1.3111028777$ and

$$m_n = \left(n + \frac{1}{2}\right) \frac{\pi}{K(i)} \left(\frac{\lambda}{2}\right)^{\frac{1}{4}} \mu. \quad (14)$$

the mass spectrum of the theory, the one of a harmonic oscillator. This can be easily seen as the Fourier transform in time at $\mathbf{p} = 0$ of this propagator has the form $\langle \phi(0, t) \phi(0, t') \rangle = \sum_{n=0}^{\infty} C_n e^{-im_n(t-t')}$ showing that, in the infrared limit, the theory develops a mass gap. This is the key result of our analysis that also completes our proof of triviality of this scalar field theory in four dimensions in the infrared limit. The final form of the generating functional at the leading order can be written down as

$$Z_0[j] = N \exp \left[\frac{i}{2} \int d^4x d^4y j(x) \Delta(x - y) j(y) \right] \quad (15)$$

having the expected Gaussian form. We observe that the free excitation we have found in this limit entails a subset of excited states with the spectrum given by eq.(14). For the

sake of completeness, we give here the next to leading order correction to this generating functional

$$Z[j] \approx Z_0[j] \int [d\phi] \exp \left\{ \frac{i}{\lambda} \int d^4x \left[\frac{1}{2}(\partial\phi)^2 - \frac{3}{2}\lambda \left(\int d^4x_1 \Delta(x-x_1) j(x_1) \right) \phi^2 \right] \right\}. \quad (16)$$

B. Yang-Mills theory

Firstly, we review an approach devised in the eighties [30, 31] that gives a clear understanding of the stumbling block arisen in the studies of infrared Yang-Mills theory. Quantum Yang-Mills theory can be stated in its simplest form through the following generating integral

$$Z[j, \epsilon, \bar{\epsilon}] = N \exp \left\{ -i \int d^4x \left[\frac{1}{4} \text{Tr} F^2 + \frac{1}{2\xi} (\partial \cdot A)^2 + (\bar{c}^a \partial_\mu \partial^\mu c^a + g \bar{c}^a f^{abc} \partial_\mu A^{b\mu} c^c) \right] \right\} \times \\ \exp \left[i \int d^4x \left(j_\mu^a A^{\mu a} + \bar{\epsilon}^a c^a + \bar{c}^a \epsilon^a \right) \right] \quad (17)$$

with the field strength given by $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$ and A_μ^a the vector potential. Now, we can proceed as for the scalar field. In this case we just rescale $x \rightarrow \sqrt{N}gx$ being N the number of colors. Then, in order to understand the behavior of Yang-Mills theory in the infrared limit, we need a way to manage the classical equations of motion (with standard notation)

$$D^\mu F_{\mu\nu}^a = -j_\nu^a \quad (18)$$

being $D_\mu = \partial_\mu - igt^a A_\mu^a$. Indeed, we can write down the solution in the form

$$A_\mu^a(x) = \Lambda \int d^4y D_{\mu\nu}^{ab}(x-y) j^{\nu b}(y) + O(1/\sqrt{N}g) \quad (19)$$

being Λ a constant having the dimension of a mass and this asymptotic approximation will be clearer in a moment. In this way we recover a Gaussian approximation at the leading order and a trivial fixed point at infrared. But at this stage we have only a conjecture unless we are able to put forward the exact form of the gluon propagator in the infrared limit, the stumbling block we talked about above. This question can be approached through a mapping theorem proved recently [32, 33] that can be stated in the following form:

Theorem 1 (Mapping) *An extremum of the action*

$$S = \int d^4x \left[\frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{4}\phi^4 \right] \quad (20)$$

is also an extremum of the $SU(N)$ Yang-Mills Lagrangian when one properly chooses A_μ^a with some components being zero and all others being equal, and $\lambda = Ng^2$, being g the coupling constant of the Yang-Mills field, when only time dependence is retained. In the most general case the following mapping holds

$$A_\mu^a(x) = \eta_\mu^a \phi(x) + O(1/\sqrt{N}g) \quad (21)$$

being η_μ^a constant, that becomes exact for the Lorenz gauge.

The proof of this theorem was completed in Ref.[33] to answer a criticism by Terence Tao. Tao finally agreed with this conclusion[34]. Stated otherwise, this theorem determines an asymptotic mapping between the solutions of the two classical theories when the couplings are taken large enough. An incomplete form of this mapping was already stated in Ref.[35]. The existence of this mapping grants that we can write down the propagator of Yang-Mills theory, e.g. in the Landau gauge, as

$$D_{\mu\nu}^{ab}(p) = \delta_{ab} \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \Delta(p) + O\left(\frac{1}{\sqrt{N}g}\right) \quad (22)$$

with $\Delta(p)$ given by eq.(12), provided we change $\lambda \rightarrow Ng^2$ and, for our aims, we dub the constant μ as Λ so that the spectrum of a $SU(N)$ can be written down as

$$m_n = \left(n + \frac{1}{2}\right) \frac{\pi}{K(i)} \left(\frac{Ng^2}{2}\right)^{\frac{1}{4}} \Lambda. \quad (23)$$

This result is very easy to prove from the definition of the two-point function

$$D_{\mu\nu}^{ab}(x-y) = \langle \mathcal{T} A_\mu^a(x) A_\nu^b(y) \rangle. \quad (24)$$

Applying the mapping theorem above one gets immediately the behavior of the theory in the infrared

$$\begin{aligned} D_{\mu\nu}^{ab}(x-y) &= \eta_\mu^a \eta_\nu^b \langle \mathcal{T} \phi(x) \phi(y) \rangle + O(1/\sqrt{N}g) \\ &= \eta_\mu^a \eta_\nu^b \Delta(x-y) + O(1/\sqrt{N}g) \end{aligned} \quad (25)$$

that produces the result. We note the identity for the Landau gauge as a possible Smilga's choice

$$\eta_\mu^a \eta_\nu^b = \delta_{ab} \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \quad (26)$$

being $\eta_{\mu\nu}$ the Minkowski metric.

For the ghost, we note that the mapping theorem grants its decoupling at the leading order. This can be seen immediately from the action of the Yang-Mills theory by direct substitution. So, at the infrared fixed point, the propagator of the ghost field is that of a free particle and we can write

$$G(p) = \frac{1}{p^2 + i\epsilon} + O(1/\sqrt{N}g). \quad (27)$$

Finally, for the generating functional one has

$$\begin{aligned} Z[j] = & N' \exp \left[\frac{i}{2} \int d^4x d^4y j^{\mu a}(x) D_{\mu\nu}^{ab}(x-y) j^{\nu b}(y) \right] \\ & + O\left(\frac{1}{\sqrt{N}g}\right) \end{aligned} \quad (28)$$

that takes also a Gaussian form showing that Yang-Mills theory in the infrared limit displays a trivial fixed point. This is the limit we are interested in here. We note as the spectrum of the theory at the trivial infrared fixed point, given by eq.(23), is made of free massive excitations, with a superimposed spectrum of a harmonic oscillator, and so entails a mass gap. These can be considered as asymptotic states to start from to build up a perturbation theory in the infrared limit. This same conclusion does not hold in QCD as this theory has a non-trivial infrared fixed point due to the presence of quarks.

III. COMPARISON WITH NUMERICAL COMPUTATIONS

In this section we show how sound is the choice of the propagator describing low-energy physics starting from measurements obtained from lattice computations and numerical solution of Dyson-Schwinger equations. This kind of computations, relatively to the Landau gauge, span a two decade long period that has seen its main breakthrough on 2007 with the clear evidence that the gluon propagator in the Landau gauge is sitting on a plateau at very low-energy, reaching a finite non-zero value at zero momenta [7–9] as also was proven in [11] for numerical Dyson-Schwinger equations. This means that, in order to show the soundness of our results given in the preceding sections, we have to compare with these computations.

We consider two kind of lattice computations: A set of volumes till 80^4 directly obtained with measurements on the lattice and measurements at 128^4 recovered from figure 2 in [8]. We are able to show in this way that, increasing the volume, our propagator describes even

more accurately the one measured on the lattice in the deep infrared. We would like to point out that the mass gap is different for these two cases as it depends on the value of β . Then, using numerical Dyson-Schwinger results [13] where no volume problem arises, we see that our propagator perfectly matches the numerical solution in the deep infrared and deviates from it in the intermediate regime where our approximation is expected to worsen. Note that we consider a weak dependence on the gauge group as showed in [36] that is fully consistent with our discussion above.

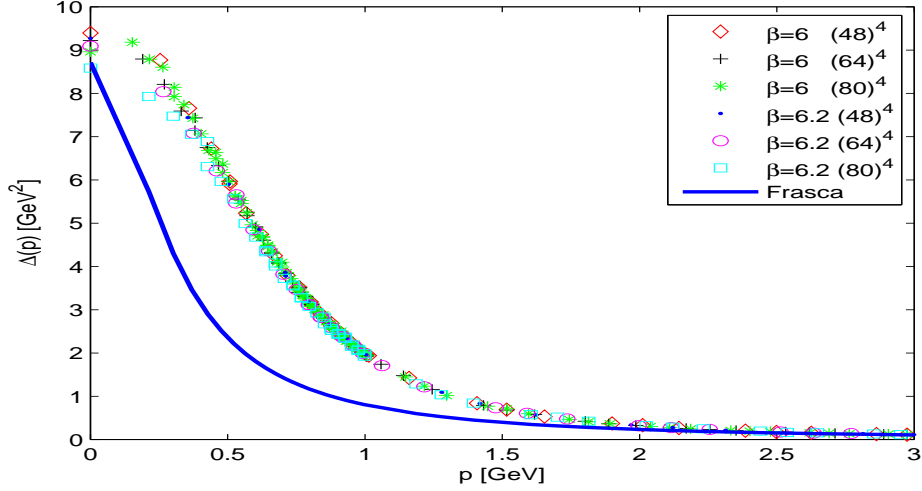


FIG. 1: Gluon propagator in the Landau gauge for SU(3), 80^4 with a mass gap of $m_0 = 287$ MeV

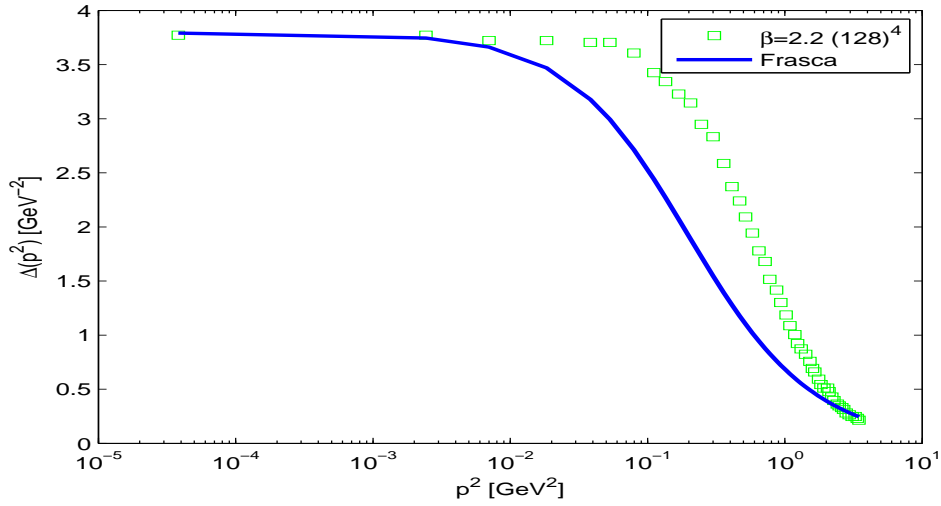


FIG. 2: Gluon propagator in the Landau gauge for SU(2), 128^4 with a mass gap of $m_0 = 435$ MeV

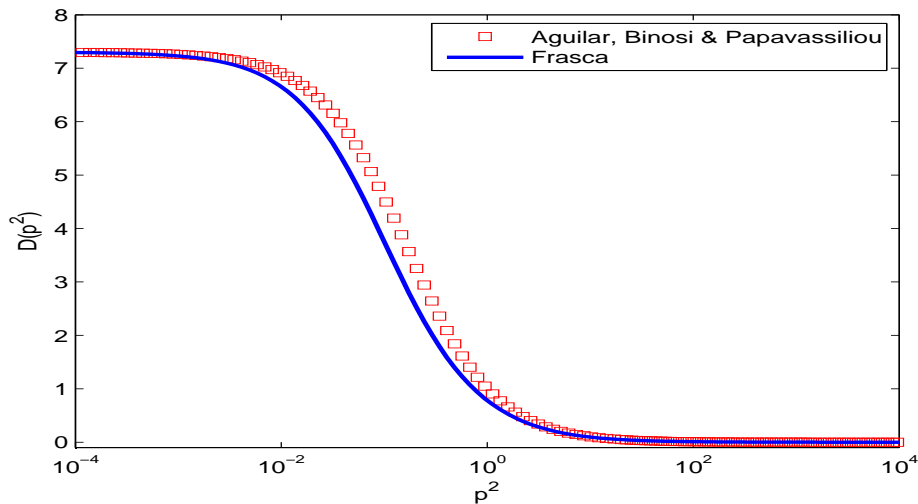


FIG. 3: Gluon propagator in the Landau gauge for SU(3) obtained by numerically solving Dyson-Schwinger equations and a mass gap $m_0 = 314$ MeV

This agreement between lattice computations at increasing volume and the perfect match for the numerical Dyson-Schwinger equations with our propagator give a strong support to the idea that a pure Yang-Mills theory reaches an infrared trivial fixed point. This same conclusion cannot be drawn for QCD due to the presence of quarks.

IV. EFFECTIVE POTENTIAL

In order to compute the effective potential for the Casimir effect, we consider the simplest geometrical setting. We assume infinite plane boundaries for the axes x, y and periodic boundary condition on the z coordinate. In our case, it is not difficult to write down the expected effective potential having an action made up of a sum of free contributions.

A. Scalar field theory

The action has the very simple form in Euclidean metric

$$S = -\frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} j(p) \Delta(p) j(-p) = \sum_{n=0}^{\infty} B_n \int \frac{d^4 p}{(2\pi)^4} j(p) \frac{1}{p^2 + m_n^2} j(-p) \quad (29)$$

showing a sum of weighted free propagators. Now, this gives us the effective action

$$\Gamma = -\frac{1}{2} \sum_{n=0}^{\infty} B_n \log \det(p^2 + m_n^2) \quad (30)$$

and using the standard relation $\ln \det = \text{Tr} \log$, with the given boundary conditions one has

$$-\frac{\Gamma}{T} = E_0 = \frac{1}{2} \sum_{n=0}^{\infty} B_n \int \frac{dp_0 dp_1 dp_2}{(2\pi)^3} \sum_{n_z=-\infty}^{\infty} \log \left(p_0^2 + p_1^2 + p_2^2 + \left(\frac{2n_z \pi}{L} \right)^2 + m_n^2 \right) S \quad (31)$$

being L the distance between the two slabs, S the surface of the boundary. In order to get the Casimir contribution we have to evaluate this expression. Firstly, we note that the sum on n_z is well-known in thermal field theory as a Matsubara sum [37]. So, one has

$$\frac{1}{2} \sum_{n_z=-\infty}^{\infty} \log \left(\omega_n^2 + \left(\frac{2n_z \pi}{L} \right)^2 \right) = \frac{L\omega_n}{2} + \ln(1 - e^{-L\omega_n}) + \text{constant} \quad (32)$$

being $\omega_n = \sqrt{p_0^2 + p_1^2 + p_2^2 + m_n^2}$ and the constant is divergent but independent on L or ω . Then, the Casimir term is straightforwardly obtained as [2]

$$E_C(L) = \sum_{n=0}^{\infty} B_n \int \frac{d^3 p}{(2\pi)^3} \ln(1 - e^{-L\omega_n}) S. \quad (33)$$

This is the standard result if there is no mass spectrum and no mass gap, that is $m_n \rightarrow 0$. In the infrared limit, being m_n finite, we have to pursue a different approach. We should consider small momenta and a cut-off μ , the same as in the mass spectrum. Then, we are able to evaluate the integral. One has

$$E_C(L) \approx - \sum_{n=0}^{\infty} B_n e^{-Lm_n} \frac{\mu^3}{6\pi^2} S \approx -B_0 e^{-Lm_0} \frac{\mu^3}{6\pi^2} S \quad (34)$$

where the approximation $Lm_0 \gg 1$ for the mass gap has been used. We can conclude from this equation that, in the infrared limit, the Casimir contribution is exponentially small. This result should be expected from the simple fact that the appearance of a mass gap makes forces short-ranged.

B. Yang-Mills theory

Due to the existence of a mapping theorem between scalar and Yang-Mills fields we can draw a similar conclusion for a gluonic field. This conclusion can change when either quarks are present or a mass gap goes to zero increasing momenta scale recovering asymptotic

freedom. This makes interesting a study at finite temperature with given boundaries. So, taking at the infrared trivial fixed point the action

$$S_0 = -\frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} j^{\mu a}(p) D_{\mu\nu}^{ab}(p) j^{\nu b}(-p) \quad (35)$$

being

$$D_{\mu\nu}^{ab}(x-y) = \delta_{ab} \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \Delta(p) \quad (36)$$

the gluon propagator in the Landau gauge and $\Delta(p)$ the propagator of the scalar field. But if we use current conservation the action is immediately reduced to

$$S_0 = -\frac{1}{2} \int \frac{d^4 p}{(2\pi)^4} j^{\mu a}(p) \Delta(p) j_\mu^a(-p) \quad (37)$$

that maps perfectly on the action of the scalar field provided a multiplying factor $d(N^2 - 1)$ that is exactly the number of current products. This means that the result is identical to the one of the scalar field apart an inessential numerical factor and we can conclude that the Casimir effect is exponential small also in Yang-Mills theory as expected.

V. CONCLUSIONS

Using recent analyses, both theoretical and numerical ones, on the behavior of the propagators of quantum field theories in the infrared limit made quite simple to derive the behavior of the corresponding vacuum in presence of a simple boundary. In the deep infrared we must expect a quite different behavior for a Yang-Mills theory due to a non-trivial vacuum, generally non-perturbatively well-described by a liquid of instantons, due to the appearance of a mass gap in the theory. A massive scalar field in the free case shows a Casimir contribution exponentially small. Here, we can see the same phenomenon to appear as both a massless scalar field theory and Yang-Mills theory acquire mass dynamically displaying a mass gap. But the situation and even more better due to the fact that these theories hit a trivial infrared fixed point making for us very easy to adopt the approaches for free theories. In the near future will be very interesting to extend this analysis to more complex boundaries and include quark matter.

Acknowledgments

I would like to thank Orlando Oliveira for providing me his measurements of the gluon propagator on the lattice, Arlene Aguilar and Daniele Binosi for providing me the numerical results of their work on Dyson-Schwinger equations. Their help speeded up a lot the completion of this work.

-
- [1] H. B. G. Casimir, *Indag. Math.* **10**, 261-263 (1948).
 - [2] M. Bordag, U. Mohideen, V. M. Mostepanenko, *Phys. Rept.* **353**, 1-205 (2001). [quant-ph/0106045].
 - [3] T. Schafer, E. V. Shuryak, *Rev. Mod. Phys.* **70**, 323-426 (1998). [hep-ph/9610451].
 - [4] L. von Smekal, R. Alkofer, A. Hauck, *Phys. Rev. Lett.* **79**, 3591-3594 (1997). [hep-ph/9705242].
 - [5] L. von Smekal, A. Hauck, R. Alkofer, *Annals Phys.* **267**, 1 (1998). [hep-ph/9707327].
 - [6] D. Zwanziger, *Nucl. Phys.* **B364**, 127-161 (1991).
 - [7] I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck, *PoS LAT2007*, 290 (2007). [arXiv:0710.1968 [hep-lat]].
 - [8] A. Cucchieri, T. Mendes, *PoS LAT2007*, 297 (2007). [arXiv:0710.0412 [hep-lat]].
 - [9] O. Oliveira, P. J. Silva, E. M. Ilgenfritz, A. Sternbeck, *PoS LAT2007*, 323 (2007). [arXiv:0710.1424 [hep-lat]].
 - [10] J. M. Cornwall, *Phys. Rev.* **D26**, 1453 (1982).
 - [11] A. C. Aguilar, A. A. Natale, *JHEP* **0408**, 057 (2004). [hep-ph/0408254].
 - [12] A. C. Aguilar, J. Papavassiliou, *JHEP* **0612**, 012 (2006). [hep-ph/0610040].
 - [13] A. C. Aguilar, D. Binosi, J. Papavassiliou, *Phys. Rev.* **D78**, 025010 (2008). [arXiv:0802.1870 [hep-ph]].
 - [14] A. C. Aguilar, D. Binosi, J. Papavassiliou, [arXiv:1107.3968 [hep-ph]].
 - [15] A. C. Aguilar, D. Binosi, J. Papavassiliou, [arXiv:1108.5989 [hep-ph]].
 - [16] D. Binosi and J. Papavassiliou, *Phys. Rept.* **479**, 1-152 (2009).
 - [17] J. Rodriguez-Quintero, *Phys. Rev.* **D83**, 097501 (2011).

- [18] J. Rodriguez-Quintero, JHEP **1101**, 105 (2011).
- [19] Ph. Boucaud, M. E. Gomez, J. P. Leroy, A. Le Yaouanc, J. Micheli, O. Pene, J. Rodriguez-Quintero, Phys. Rev. **D82**, 054007 (2010).
- [20] Ph. Boucaud, F. De Soto, J. P. Leroy, A. Le Yaouanc, J. Micheli, O. Pene and J. Rodriguez-Quintero, Phys. Rev. D **79**, 014508 (2009).
- [21] Ph. Boucaud, J. P. Leroy, A. L. Yaouanc, J. Micheli, O. Pene and J. Rodriguez-Quintero, JHEP **0806** (2008) 012.
- [22] M. Frasca, Phys. Rev. D **73**, 027701 (2006) [Erratum-ibid. D **73**, 049902 (2006)] [arXiv:hep-th/0511068].
- [23] M. Frasca, arXiv:0802.1183 [hep-th].
- [24] M. Frasca, PoS **FACESQCD**, 039 (2011). [arXiv:1011.3643 [hep-th]].
- [25] C. M. Bender, F. Cooper, G. S. Guralnik and D. H. Sharp, Phys. Rev. D **19**, 1865 (1979).
- [26] C. M. Bender, F. Cooper, G. S. Guralnik, R. Roskies and D. H. Sharp, Phys. Rev. Lett. **43**, 537 (1979).
- [27] M. Frasca, Mod. Phys. Lett. A **22**, 1293 (2007) [arXiv:hep-th/0702056].
- [28] M. Frasca, Int. J. Mod. Phys. A **23**, 299 (2008) [arXiv:0704.1568 [hep-th]].
- [29] M. Frasca, Int. J. Mod. Phys. A **22**, 2433 (2007) [arXiv:hep-th/0611276].
- [30] J. T. Goldman, R. W. Haymaker, Phys. Rev. **D24**, 724 (1981).
- [31] R. T. Cahill, C. D. Roberts, Phys. Rev. **D32**, 2419 (1985).
- [32] M. Frasca, Phys. Lett. B **670**, 73 (2008) [arXiv:0709.2042 [hep-th]].
- [33] M. Frasca, Mod. Phys. Lett. A **24** (2009) 2425 [arXiv:0903.2357 [math-ph]].
- [34] T. Tao, private communication (2009) and <http://tosio.math.utoronto.ca/wiki/index.php/Talk:Yang-Mills>
- [35] A. V. Smilga, *Lectures on quantum chromodynamics*, (World Scientific, Singapore, 2001).
- [36] A. Maas, JHEP **1102**, 076 (2011). [arXiv:1012.4284 [hep-lat]].
- [37] M. Le Bellac, “Thermal Field Theory”, (Cambridge University Press, Cambridge, 1996).